

Joint Physical Network Coding and LDPC decoding for Two Way Wireless Relaying

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Abstract

In this paper, we investigate the joint design of channel and network coding in bi-directional relaying systems and propose a combined low complexity physical network coding and LDPC decoding scheme. For the same LDPC codes employed at both source nodes, we show that the relay can decode the network coded codewords from the superimposed signal received from the BPSK-modulated multiple-access channel. Simulation results shown that this novel joint physical network coding and LDPC decoding method outperforms the existing MMSE network coding and LDPC decoding method over AWGN and complex MAC channel.

Keywords: Joining Network Coding and LDPC Decoding; Two Way Wireless Relaying Channel; Wireless Cooperative Networks.

1. Introduction

The network coding scheme was originally considered as a technique of improving network throughput for wired networks [1]. In wireless network, the broadcast nature of the wireless channel is usually considered to cause enormous interference if more than two nodes transmit simultaneously at the same frequency. On the other hand, physical network coding (PNC) [2,3] can employ this broadcast nature as a capacity-boosting approach for two-way or multi-way cooperative communication network.

A simple two-way wireless relaying system with two sources A and B and one relay R is depicted in Fig.1. Source A and source B desire to exchange information between each other and there is no direct link between the two source nodes. Thus, all the transmission between source A and B must flow through the relay R. The relay transmission consists of two states: multiple access (MAC) stage, where source A and B transmit the LDPC-coded signals to the relay R simultaneously, and broadcast (BC) stage, where the relay R broadcasts to both source A and B. One critical process at R is to decode the superimposed signal from A and B at MAC stage in such a way that A and B could decode the information from each other reliably at the BC stage. Instead of decoding the individual information belonging to the source A and B

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separately, the relay node R aims to decode the received superimposed signal to the network-coded combination of the two sources information. We refer this decoding process as the joint physical network coding and LDPC decoding (JNCLD).

In [4,5] non-coherent physical layer network coding for FSK and CPFSK modulation are proposed. In [6], joint network and channel coding was proposed for the simple real additive multiple-access white Gaussian noise channel. By noticing the linearity of both network and channel coding, the soft Log-likelihood Ratios (LLRs) for the network-coded codeword can be directly estimated from the received physically superimposed signals. In [7,8], a joint network and LDPC coding scheme for bi-directional relaying is presented and the closed-form expressions for computing the log-likelihood ratios of the network-coded codewords have been derived for both real and complex multiple access channels. An adaptive PNC called pseudo exclusive-or (PXOR) for LDPC coded two-way relay block fading channels is proposed in [12]. Based on the pairwise check decoding (PCD) [13], the check relationship table generated by PXOR mapping obtains the same Hamming distances as that of conventional XOR mapping. In order to compensate the amplitude fading and phase deviation of the TWR block fading channels, the PXOR mapping optimizes the Euclidean distances by adjusting the symbol distances dynamically.

Optimal time and rate allocation scheme for a network-coded bidirectional wireless communication system is proposed in [14]. The closed form expressions for the optimal allocation of the transmission time and of the data rate in both ways for given channel SNRs to maximizing the sum-rate is derived. In [9,10], a joint PNC and minimum mean square error (MMSE) based LDPC decoding method is proposed. In this paper, we provide some novel insights into the decode-and-forward approach for two way wireless relaying with BPSK signaling. In particular, for the same LDPC codes employed at both source nodes, a novel iterative LDPC decoding algorithm is proposed for the physical network coding scheme at relay, which outperforms recently proposed MMSE based network coding scheme on bit error rate (BER) performance.

The remainder of this paper is organized as follows. In Section II we present the considered system model. Section III present the joint PNC and MMSE based LDPC decoding scheme [9,10] and introduce the proposed novel joint physical network coding and LDPC decoding approach. Finally, in Section IV and V simulation results are presented and conclusion is given.

2. System Model

We denote $b_A \in \{0, 1\}^K$ and $b_B \in \{0, 1\}^K$ as the information vector of the two source nodes A and B, respectively. The information is encoded by the same LDPC code with a code rate of K/N into the codeword vectors $c_A \in \{0, 1\}^N$ and $c_B \in \{0, 1\}^N$ at the sources A and B, respectively. The encoded vectors are BPSK-modulated to $x_A \in \{-1, 1\}^N$ and $x_B \in \{-1, 1\}^N$ according to the mapping rule $1 \rightarrow 1$ and $0 \rightarrow -1$.

2.1. Multiple Access Stage

During the MAC stage, the two sources transmit the modulated signals x_A and x_B to the relay R simultaneously. Under a multiple access white Gaussian noise channel and the assumption of perfect synchronization, the received signal at the relay R is

$$y_R = x_A + x_B + n_R \quad (1)$$

where n_R are identically distributed (i.i.d) zero-mean Gaussian random variables with variance σ_n^2 . According to the physical network coding scheme introduced in [6-11], the XOR of the

source information denoted by $b_{A\oplus B} = b_A \oplus b_B \in \{0, 1\}^K$ can be estimated at the relay from the received signal y_R , i.e., $b_R = \hat{b}_{A\oplus B} \in \{0, 1\}^K$. Then, b_R is encoded by the same LDPC code, and the code vector c_R BPSK-modulated to x_R .

2.2. Broadcasting Stage

In the BC stage, the relay R broadcasts x_R to both A and B. Thus, the received signals at A and B are given by

$$\begin{aligned} y_A &= x_R + n_A \\ y_B &= x_R + n_B \end{aligned} \quad (2)$$

At both A and B, the information \hat{b}_R , which contains the information of $b_{A\oplus B}$, is estimated from y_A and y_B , respectively. Since A and B know what has been transmitted at the MAC stage, A and B can obtain the information from each other simply by means of the binary XOR, i.e., $\hat{b}_B = \hat{b}_R \oplus \hat{b}_A$ and $\hat{b}_A = \hat{b}_R \oplus \hat{b}_B$.

A critical process at the relay R is to decode the superimposed signal from A and B in such a way that A and B could decode the information from each other reliably at the BC stage. In this paper, we will focus on deriving a decoding algorithm for $y_R \rightarrow b_R$.

3. Joint Physical Network Coding and LDPC Decoding

In [9,10], joint PNC and MMSE based LDPC decoding method is proposed. The idea of this algorithm is that under the assumption of the same LDPC code applied at both source nodes, i.e., parity check matrix $H_A = H_B = H$ and

$$\begin{aligned} H_A c_A &= 0, \\ H_B c_B &= 0, \\ H(c_A \oplus c_B) &= 0 \end{aligned} \quad (3)$$

the XOR of the encoded symbols $c_{A\oplus B} \in \{0, 1\}^N$ is also a valid codeword of the LDPC code. The relay R first maps each pair of the received superimposed signal y_R to an estimation of the joint network and LDPC coded symbol \hat{y}_R^{MMSE} corresponding to $c_A \oplus c_B$ by using minimum mean square error (MMSE) estimation; then performs LDPC decoding on the interim symbol $c_A \oplus c_B$ to obtain the network coded symbol $b_A \oplus b_B$.

Different from the above mentioned approach, we will compute the LLR $\Lambda(c_{A\oplus B})$ of the XOR of the two source information $c_A \oplus c_B$ direct from the received signal y_R and then performs LDPC decoding to obtain the network coded symbol $b_A \oplus b_B$.

3.1. AWGN MAC Channel with Power Allocation

The received superimposed signal y_R over AWGN MAC channel with power allocation can be expressed as

$$y_R = h_A \rho_A x_A + h_B \rho_B x_B + n_R \quad (4)$$

where h_A and h_B represent the channel gain from node A and B to the relay node R, respectively. $P_A = \rho_A^2$ and $P_B = \rho_B^2$ denote the power allocated to the MAC channel, respectively. Let n index the bit of a codeword. The a-priori probabilities of $\{c_{A\oplus B}(n) = c_A(n) \oplus c_B(n) = 0, 1\}$ are

$$\begin{aligned} Pr\{c_{A\oplus B}(n) = 0\} &= 1/2 \\ Pr\{c_{A\oplus B}(n) = 1\} &= 1/2 \end{aligned} \quad (5)$$

If $c_{A\oplus B}(n) = 1$ holds, the event $E_1 = \{c_A(n) = 0, c_B(n) = 1\}$ or the event $E_2 = \{c_A(n) = 1, c_B(n) = 0\}$ should be satisfied. On the other side, if $c_{A\oplus B}(n) = 0$ holds, the event $E_3 = \{c_A(n) = 1, c_B(n) = 1\}$ or the event $E_4 = \{c_A(n) = 0, c_B(n) = 0\}$ should be satisfied. Hence, the probability of $\{c_{A\oplus B}(n) = 1\}$ is the sum probability of $Pr\{E_1\}$ and $Pr\{E_2\}$, and the probability of $\{c_{A\oplus B}(n) = 0\}$ is the sum probability of $Pr\{E_3\}$ and $Pr\{E_4\}$.

Assuming that the relay has knowledge of parameter σ_R^2 and channel state information. Soft-decision decoding requires that the relay node R compute the LLR of each network coded bit $c_{A\oplus B}$ according to

$$\begin{aligned}\Lambda_{AM}(c_{A\oplus B}) &= \log \frac{P(c_{A\oplus B} = 1|y_R)}{P(c_{A\oplus B} = 0|y_R)} \\ &= \log \frac{P(c_A \oplus c_B = 1|y_R)}{P(c_A \oplus c_B = 0|y_R)} \\ &= \log[P(y_R|E_1) + P(y_R|E_2)] - \log[P(y_R|E_3) + P(y_R|E_4)]\end{aligned}\quad (6)$$

where

$$P(y_R|E_1) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{(y_R - h_A\rho_A + h_B\rho_B)^2}{2\sigma_R^2}\right\} \quad (7)$$

$$P(y_R|E_2) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{(y_R + h_A\rho_A - h_B\rho_B)^2}{2\sigma_R^2}\right\} \quad (8)$$

$$P(y_R|E_3) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{(y_R - h_A\rho_A - h_B\rho_B)^2}{2\sigma_R^2}\right\} \quad (9)$$

$$P(y_R|E_4) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{(y_R + h_A\rho_A + h_B\rho_B)^2}{2\sigma_R^2}\right\} \quad (10)$$

The LLR of the network coded bit $c_{A\oplus B}$ over AWGN MAC channel with power allocation can be expressed as

$$\begin{aligned}\Lambda_{AM}(c_{A\oplus B}) &= \frac{2h_A h_B \rho_A \rho_B}{\sigma_R^2} + \log\left(\cosh\left(\frac{y_R(h_A\rho_A - h_B\rho_B)}{\sigma_R^2}\right)\right) \\ &\quad - \log\left(\cosh\left(\frac{y_R(h_A\rho_A + h_B\rho_B)}{\sigma_R^2}\right)\right)\end{aligned}\quad (11)$$

The network coded information bit $b_{A\oplus B}$ can be obtained by the traditional belief broadcasting (BP) decoding algorithm with LLR $\Lambda_{AM}(c_{A\oplus B})$.

3.2. Complex MAC channel with Power Allocation

The received superimposed signal \tilde{y}_R over complex MAC channel with power allocation can be expressed as

$$\tilde{y}_R = h_A e^{j\theta_A} \rho_A x_A + h_B e^{j\theta_B} \rho_B x_B + n_R \quad (12)$$

where n_R denotes the zero-mean complex Gaussian noise for complex MAC channel with covariance σ_R^2 , h_A and h_B represent the channel gain from node A and B to the relay node R,

respectively. $P_A = \rho_A^2$ and $P_B = \rho_B^2$ denote the power allocated to the complex MAC channel, respectively. θ_A and θ_B represents uniformly distributed phase shift over $[0, 2\pi)$. The LLR of each network coded bit $c_{A\oplus B}$ with the assumption that the relay has knowledge of parameter σ_R^2 and channel state information can be expressed as

$$\begin{aligned}\Lambda_{CM}(c_{A\oplus B}) &= \log \frac{P(c_{A\oplus B} = 1|\tilde{y}_R)}{P(c_{A\oplus B} = 0|\tilde{y}_R)} \\ &= \log \frac{P(c_A \oplus c_B = 1|\tilde{y}_R)}{P(c_A \oplus c_B = 0|\tilde{y}_R)} \\ &= \log[P(\tilde{y}_R|E_1) + P(\tilde{y}_R|E_2)] - \log[P(\tilde{y}_R|E_3) + P(\tilde{y}_R|E_4)]\end{aligned}\quad (13)$$

where

$$P(\tilde{y}_R|E_1) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{\|\tilde{y}_R - h_A e^{j\theta_A} \rho_A + h_B e^{j\theta_B} \rho_B\|^2}{2\sigma_R^2}\right\} \quad (14)$$

$$P(\tilde{y}_R|E_2) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{\|\tilde{y}_R + h_A e^{j\theta_A} \rho_A - h_B e^{j\theta_B} \rho_B\|^2}{2\sigma_R^2}\right\} \quad (15)$$

$$P(\tilde{y}_R|E_3) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{\|\tilde{y}_R - h_A e^{j\theta_A} \rho_A - h_B e^{j\theta_B} \rho_B\|^2}{2\sigma_R^2}\right\} \quad (16)$$

$$P(\tilde{y}_R|E_4) = \frac{1}{4\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{\|\tilde{y}_R + h_A e^{j\theta_A} \rho_A + h_B e^{j\theta_B} \rho_B\|^2}{2\sigma_R^2}\right\} \quad (17)$$

The LLR of the network coded bit $c_{A\oplus B}$ over complex MAC channel with power allocation can be expressed as

$$\begin{aligned}\Lambda_{AM}(c_{A\oplus B}) &= \frac{\|h_A e^{j\theta_A} \rho_A + h_B e^{j\theta_B} \rho_B\|^2 - \|h_A e^{j\theta_A} \rho_A - h_B e^{j\theta_B} \rho_B\|^2}{\sigma_R^2} \\ &+ \log\left(\cosh\left(\frac{\Re(\tilde{y}_R(h_A e^{j\theta_A} \rho_A - h_B e^{j\theta_B} \rho_B)^*)}{\sigma_R^2}\right)\right) \\ &- \log\left(\cosh\left(\frac{\Re(\tilde{y}_R(h_A e^{j\theta_A} \rho_A + h_B e^{j\theta_B} \rho_B)^*)}{\sigma_R^2}\right)\right)\end{aligned}\quad (18)$$

The network coded information bit $b_{A\oplus B}$ can be obtained by the traditional BP decoding algorithm with LLR $\Lambda_{CM}(c_{A\oplus B})$.

4. Simulation Results

In this section, we demonstrate the simulated performance of the proposed joint physical network coding and LDPC decoding scheme for two way wireless relay systems with power allocation in each source node. In the simulation, we assume that the SNR at the relay node R is defined as $(P_A h_A^2 + P_B h_B^2)/\sigma_R^2$ for both AWGN and complex MAC channel, where σ_R^2 denotes the noise variance received at the relay R and the total transmitting power $P_A h_A^2 + P_B h_B^2$ is set to

2. We check the BER (bit error rate) of the decoded packet $b_{A\oplus B} = b_A \oplus b_B$ at the relay node R. BPSK modulation scheme is used at both end nodes for all simulations.

For comparison, we also study the performance of MMSE based method proposed in [9,10]. We first compare the BER performance of MMSE estimation and the proposed method over AWGN MAC channel with power allocation for various packet length. In Fig. 2, the ration of $P_A h_A^2 / P_B h_B^2$ is set to be 2/3 and the iteration numbers of the two schemes are set to 30. The proposed method outperforms MMSE about 0.2dB when the BER is 10^{-4} for the packet length (PL) of 1010 over AWGN MAC channel. The BER performance of MMSE estimation and the proposed method over complex MAC channel is given in Fig. 3, the power ration of the two source nodes $P_A h_A^2 / P_B h_B^2$ is also set to be 2/3. We can see from Fig.3 that the proposed method outperforms MMSE based method about 0.1dB when the BER is 10^{-4} for the PL of 1010.

5. Conclusion

In this paper, a novel joint physical network coding and LDPC decoding method for two way wireless relaying system with BPSK signaling is presented. The proposed method employ the same LDPC codes at the source nodes and the relay decodes the network coded packet from received superimposed signal by using the proposed method. Simulation results shown that the proposed novel method outperforms MMSE method about 0.2dB and 0.1dB over AWGN and complex MAC channel, respectively.

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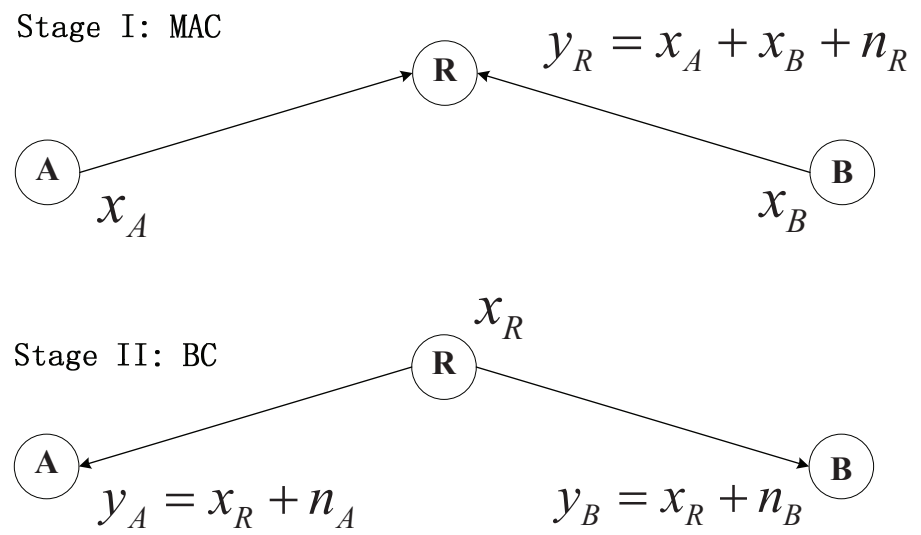


Figure 1: Two way relay channel

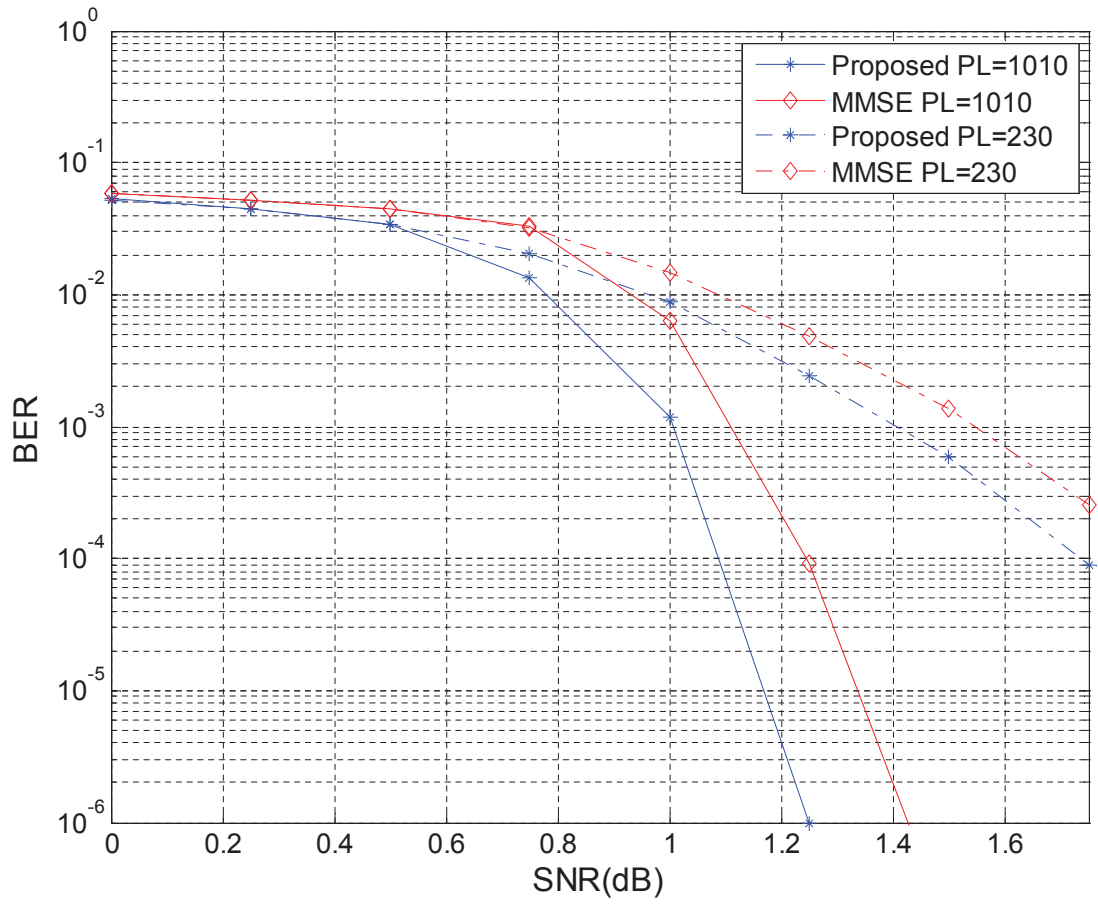


Figure 2: BER performance of the proposed method and MMSE method for various packet lengths and power allocation over AWGN MAC channel

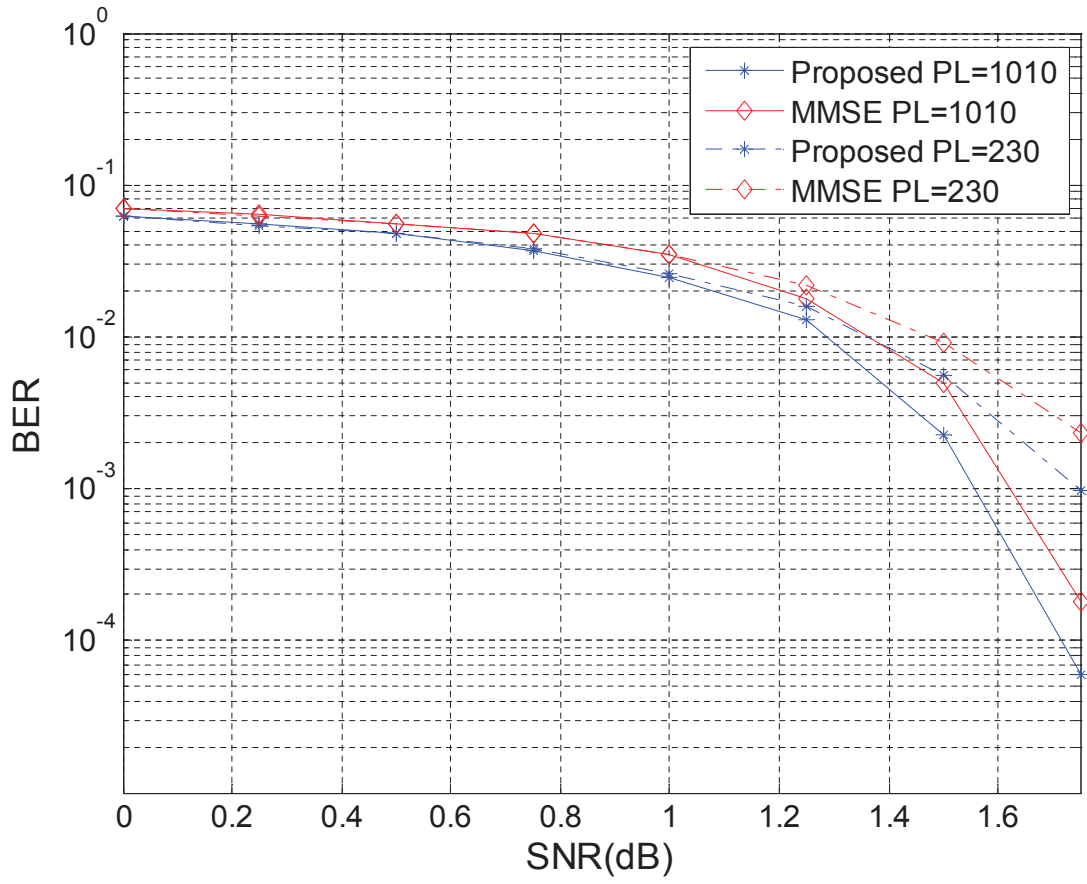


Figure 3: BER performance of the proposed method and MMSE method for various packet lengths with power allocation and uniformly distributed phase shift caused by complex MAC channel